Public-key Cryptography

What Is Cryptography?

- Cryptography -- from the Greek for "secret writing" -- is the mathematical "scrambling" of data so that only someone with the necessary ***key*** can "unscramble" it.
- Cryptography allows secure transmission of private information over insecure channels (for example packet-switched networks).
- Cryptography also allows secure storage of sensitive data on any

Classical Cryptography:

Secret-Key or Symmetric Cryptography

- Alice and Bob agree on an encryption method and a shared ***key***.
- Alice uses the key and the encryption method to ***encrypt*** (or ***encipher***) a message and sends it to Bob.
- Bob uses the same key and the related decryption method to ***decrypt*** (or ***decipher***) the message.

Advantages of Classical Cryptography

- There are some very fast classical encryption (and decryption) algorithms
- Since the speed of a method varies with the length of the key, faster algorithms allow one to use longer key values.
- Larger key values make it harder to guess the key value -- and break the code -- by brute force.

Disadvantages of Classical Cryptography

- ***Requires secure transmission of key value***
- Requires a separate key for each group of people that wishes to exchange encrypted messages (readable by any group member)
  - For example, to have a separate key for each pair of people, 100 people would need 4950 different keys.

Public-Key Cryptography: Asymmetric Cryptography

- Alice generates a key value (usually a number or pair of related numbers) which she makes public.
- Alice uses her public key (and some additional information) to determine a second key (her ***private key***).
- Alice keeps her private key (and the additional information she used to construct it) secret.

Public-Key Cryptography (continued)

- Bob (or Carol, or anyone else) can use Alice’s public key to encrypt a message for Alice.
- Alice can use her private key to decrypt this message.
- No-one without access to Alice’s private key (or the information used to construct it) can easily decrypt the message.

An Example: Internet Commerce

- Bob wants to use his credit card to buy some brownies from Alice over the Internet.
- Alice sends her public key to Bob.
- Bob uses this key to encrypt his credit-card number and sends the encrypted number to Alice.
- Alice uses her private key to decrypt this message (and get Bob’s credit-card number).

Hybrid Encryption Systems

- All known public key encryption algorithms are much slower than the fastest secret-key algorithms.
- In a ***hybrid*** system, Alice uses Bob's public key to send him a secret shared ***session key***.
- Alice and Bob use the session key to exchange information.

Internet Commerce (continued)

- Bob wants to order brownies from Alice and keep the ***entire transaction*** private.
- Bob sends Alice his public key.
- Alice generates a session key, encrypts it using Bob's public key, and sends it to Bob.
- Bob uses the session key (and an agreed-upon symmetric encryption algorithm) to encrypt his order, and

Digital Signatures:
Signing a Document

- Alice applies a (publicly known) ***hash function*** to a document that she wishes to “sign.” This function produces a ***digest*** of the document (usually a number).
- Alice then uses her ***private*** key to “encrypt” the digest.
- She can then send, or even broadcast, the document with the encrypted digest.

Digital Signature Verification

- Bob uses Alice’s ***public*** key to “decrypt” the digest that Alice “encrypted” with her private key.
- Bob applies the hash function to the document to obtain the digest directly.
- Bob compares these two values for the digest. If they match, it proves that Alice signed the document and that no one else has altered it.

Secure Transmission of Digitally Signed Documents

- Alice uses her ***private*** key to digitally sign a document. She then uses Bob's ***public*** key to encrypt this digitally signed document.
- Bob uses his ***private*** key to decrypt the document. The result is Alice's digitally signed document.
- Bob uses Alice's ***public*** key to verify Alice's digital signature.

Historical Background

- 1976: W. Diffie and M.E. Hellman proposed the first public-key encryption algorithms -- actually an algorithm for public ***exchange*** of a secret key.
- 1978: L.M Adleman, R.L. Rivest and A. Shamir propose the RSA encryption method
  - Currently the most widely used
  - Basis for the spreadsheet used in the lab

The RSA Encryption Algorithm

- Use a random process to select two large prime numbers ***P*** and ***Q***. Compute the product ***M = P*Q***. This number is called the ***modulus***, and is made publicly available.
  - RSA currently recommends a modulus that's at least 768 bits long.
- Also compute the ***Euler totient*** ***T = (P-1)*(Q-1)***. Keep this number (as well as ***P*** and ***Q***) secret.

RSA (continued)

- Randomly choose a public key ***E*** that has no factors in common with ***T = (P-1)*(Q-1)***.
- Compute a private key ***D*** so that ***E*D*** leaves a remainder of 1 when divided by ***T***.
  - We say ***E*D*** is ***congruent*** to 1 ***modulo T***
- Note that ***D*** is easy to compute only if one knows the value of T. This is essentially the same as knowing the

RSA (continued)

- If ***N*** is any number that is not divisible by ***M***, then dividing N^{E*D} by ***M*** and taking the remainder yields the original value ***N***.
  - This is a relatively deep mathematical theorem, which we can write as $N^{E*D} \bmod M = N$.)
- If ***N*** is a numeric encoding of a block of plaintext, the cyphertext ***is*** $C = N^E \bmod M$.
- Then $C^D \bmod M = (N^E)^D \bmod M = N^{E*D} \bmod M = N$. Thus, we can

Why RSA Works

- Multiplying P by Q is ***easy***: the number of operations depends on the ***number of bits*** (number of digits) in P and Q.
- For example, multiplying two 384-bit numbers takes approximately $384^2 = 147{,}456$ bit operations

Why RSA Works (2)

- If one knows only M, finding P and Q is ***hard***: in essence, the number of operations depends on the ***value*** of M.
  - The simplest method for factoring a 768-bit number takes about $2^{384} \approx 3.94 \times 10^{115}$ trial divisions.
  - A more sophisticated methods takes about $2^{85} \approx 3.87 \times 10^{25}$ trial divisions.
  - A still more sophisticated method takes about $2^{41} \approx 219{,}000{,}000{,}000$ trial divisions

Why RSA Works (3)

- No-one has found an really quick algorithm for factoring a large number ***M***.
- No-one has proven that such a quick algorithm doesn’t exist (or even that one is unlikely to exist).
- Peter Shor has devised a very fast factoring algorithm for a ***quantum computer***, if anyone manages to build one.

Error Detection

- There will always be errors
- How to measure the errors - Bit Error Rate (BER)
  - Probability of an error
- Single and burst errors

- Error Detection
  - For a given frame of bits, additional bits that constitute an error-detecting code are added by the transmitter
  - The code is calculated as a function of the other transmitted bits
  - The receiver performs the same calculation and compares the results

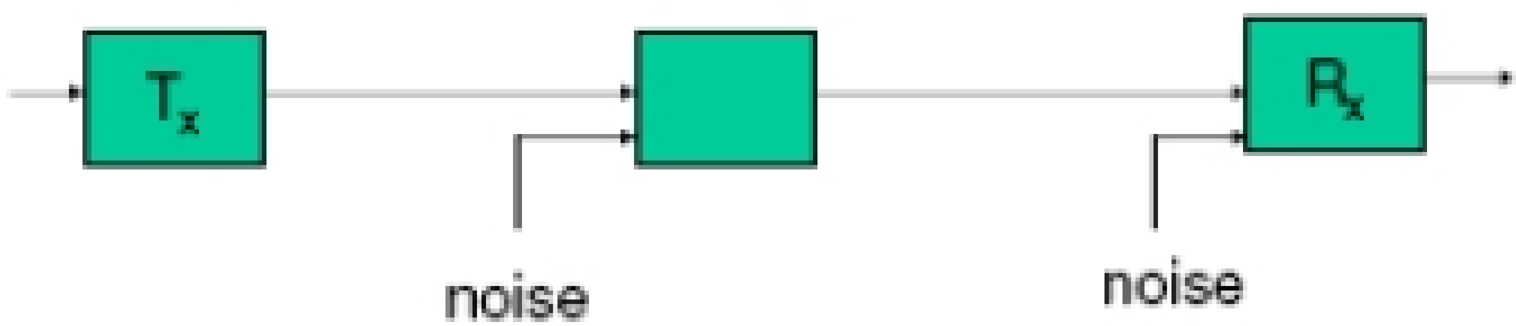
Error Detection

- There will always be errors
- How to measure the errors - Bit Error Rate (BER)
  - Probability of an error
- Single and burst errors

- Error Detection
  - For a given frame of bits, additional bits that constitute an error-detecting code are added by the transmitter
  - The code is calculated as a function of the other transmitted bits
  - The receiver performs the same calculation and compares the results

Error Accumulation over multiple hops

- Attenuation / Regeneration
- Amplifiers vs. repeaters
- Errors accumulate over multiple hops
- Regeneration is necessary



Error Detection Methods

- Parity
- Block Sum Check
- Cyclic Redundancy Check

Parity

- Value of parity bit is such that character has even (even parity) or odd (odd parity) number of 1's
- Even number of bit errors is not detected

<u>Example:</u>

1001001 1 (Even Parity)

1001001 0 (Odd Parity)

Block Sum Check

Row parity bits (odd)

P_r	B_6	B_5	B_4	B_3	B_2	B_1	B_0
0	0	0	0	0	0	1	0
1	0	1	0	1	0	0	0
0	1	0	0	0	1	1	0
0	0	1	0	0	0	0	0
1	0	1	0	1	1	0	1
0	1	0	0	0	0	0	0
1	1	1	0	0	0	1	1
1	0	0	0	0	0	1	1
1	1	0	0	0	0	0	1

Column parity bits (even)

transmission

Block Sum Check

- Each character in the block is assigned a parity bit
- In addition, a parity bit is calculated across all the characters - one for each bit position
- The resulting set of parity bits is the block check character
- Detects multiple errors

Cyclic Redundancy Check - Polynomial Codes

- Detecting strings of errors
- For a block of *k* bits transmitter generates *n* bit sequence
- Transmit *k+n* bits which is exactly divisible by some number
- Receive divides frame by the same number
  - If no remainder, assume no error

Error Correction

- The loss of even a single bit can have potentially catastrophic consequences
- Two basic approaches :
  - **Forward Error Control** (there is additional info in each character which can help the receiver to create the correct data)
  - **Feedback Error Control** (additional info detects the error and then retransmission scheme is deployed)

Error Correction

- Forward Error Correction
  - Rarely used in data transmission
  - Used when retransmission is not practical – broadcast transmission
  - The receiver is correcting the code
  - Example – Hamming code

- Feedback Error Correction
  - Described before
  - Automatic Repeat Request (ARQ): Stop and Wait, Go Back N, Selective Repeat